Chapter 5

PHASE ABERRATIONS IN BRAGG IMAGING

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ABSTRACT

Theoretical investigations of Bragg imaging in three dimensions show that a cylindrically convergent light beam reconstructs each plane wave component of the sound field unambiguously. However, aberrations in the image plane arise for those sound components that project out of the plane normal to the line formed by the light. Particularly it is shown that images from long objects oriented parallel to the convergence of the light beam are free from significant phase aberrations (i.e., the components at the image plane have essentially the same phase with respect to each other as the associated sound components). However, similar analysis and experiments show that images of objects oriented at right angles to this direction do indeed have a significant phase aberration.

INTRODUCTION

Bragg imaging is a means for optically imaging an acoustic field.[1,2,3] In this method, interaction between light and sound serves to diffract a portion of a laser beam passing through a sound field in the interaction medium. If the laser beam is properly shaped before it enters the sound field, this diffracted light will then replicate the sound pattern and be available for viewing and imaging with optical devices.

Figure 1 illustrates a Bragg imaging system in a configuration to obtain a transmission type image (i.e., an image from sound which passes through an object). In this arrangement the acoustic field insonifying the object is generated by a quartz transducer driven by the rf generator and amplifier. The sound field containing the object information then interacts with the laser light. The laser beam is in the form of a wedge with the apex located to the right of the interaction, as in the figure. A vertically oriented cylindrical lens to the left of the acoustic cell acts on a collimated laser beam to produce the wedge shape. (The plane of the figure is assumed to be the horizontal plane.) At approximately the plane P_1, two images of the sound field are formed, one to either side of the central-order light. The image to the left (looking along the axis of light propagation) is an upshifted (in frequency) virtual image of the cross section of the sound field. The image to the right is a downshifted real image. Both images are demagnified in the horizontal direction by the ratio of the light wavelength to the sound wavelength. For rf sound frequencies and He-Ne laser light, this demagnification is of the order of 10^{-2}. The vertically oriented cylindrical projecting lens focuses the vertical features of one of these images, the downshifted real image, onto the television camera face, restoring the horizontal dimension of the image to a useful size. At plane P_2 a stop removes the central-order beam and the other image. The remaining image is then focused as far as its horizontal features are concerned by the horizontally oriented cylindrical lens. Thus an in-focus real image of the sound field is projected directly onto a vid:con tube for television display. Images can then ,shown on the screen for real-time viewing or f photographic recording.

The choice of a wedge shape for the laser beam (or equivalently, a line focus for the beam) is

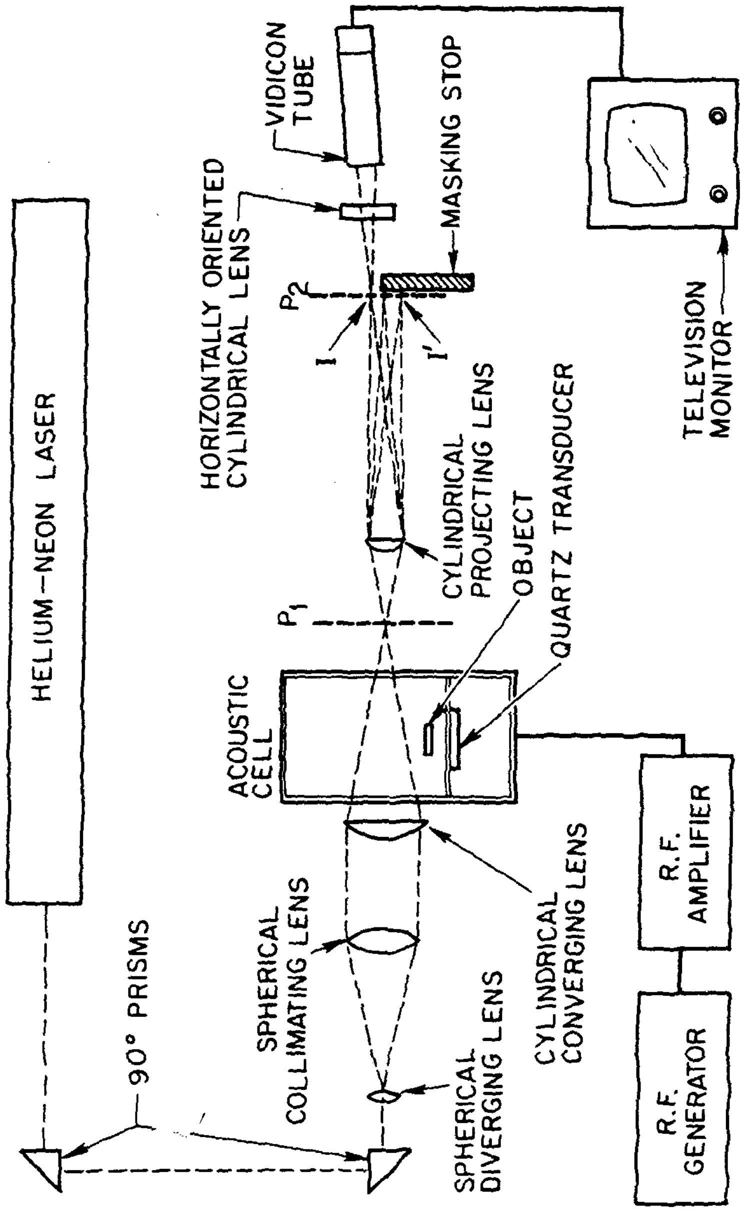

Fig. 1 - A Bragg imaging system operating in the transmission mode.

made in order to provide the best image. Theoretical analysis[1] using wave theory has verified that this simple configuration of the light does provide better images than some other configurations. Further investigation[3] based on angular spectrum arguments has shown that the diffracted light reproduces the angular spectrum of the sound on a component-by-component basis. This unambiguous reconstruction of the angular spectrum does not, by itself, guarantee an undistorted image of the sound field. Aberrations of various kinds can still be present. One such aberration, namely phase aberration, is the subject of this investigation.

In this paper we will be concerned only with the downshifted real image. However, a similar study could be made for the upshifted virtual image. The method of analysis will be based on a Fourier decomposition[4] of the fields involving the interacting light, the sound and the diffracted light. Each of these fields is decomposed into an infinite set of plane waves of infinite extent. This decomposition may be represented mathematically as

$$\underline{U}(x,y) = \iint_{-\infty}^{\infty} \underline{U}'(f_x,f_y)e^{-j2\pi(f_x x+f_y y)}\,df_x df_y$$

where $\underline{U}(x,y)$ is the cross section of a complex-valued scalar wave field propagating in the +z direction and $\underline{U}'(f_x,f_y)$ specifies the amplitude and phase of each plane-wave component whose propagation direction is specified by the arguments f_x and f_y. (The direction cosines of the propagation direction with respect to the x or y axis are equal to the wavelength times the value of f_x or f_y, respectively.) The planar-wave component $\underline{U}'(f_x,f_y)$ may be evaluated by taking the spatial Fourier transform of the scalar field across the cross section:

$$\underline{U}'(f_x,f_y) = \iint_{-\infty}^{\infty} \underline{U}(x,y)e^{-j2\pi(f_x x+f_y y)}\,dxdy$$

The Fourier decomposition can then be applied to the imaging process as follows: The incident light field and sound field are expressed as plane-wave components. All components that meet the required angular conditions for Bragg diffraction interact to produce a diffracted component. For the downshifted case the propagation vectors of the interacting plane waves must intersect at an angle of $\pi/2-\phi_B$. Here ϕ_B is the Bragg angle and is specified by

$$\sin \phi_B = \frac{\lambda}{2\Lambda}$$

where λ is the wavelength of the incident light and Λ is the wavelength of the sound. The diffracted component will have a propagation vector which is directed $\pi/2-\phi_B$ radians away from the sound component on the side opposite the incident light. An illustration of this criterion is shown in Fig. 2.

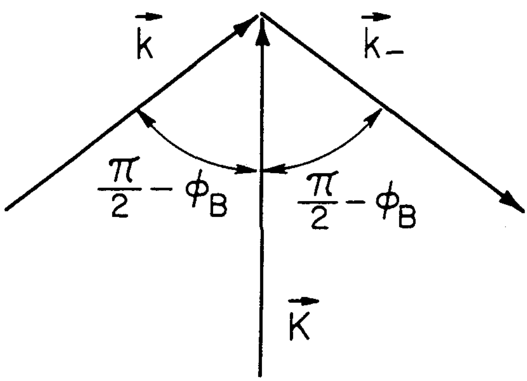

Fig. 2 - Orientation of propagation vectors in the case of downshifted Bragg diffraction.

The amplitude of the diffracted plane-wave component is equal to the product of the amplitudes of the interacting components. The phase of the diffracted component is given by the difference in phase of the interacting components. The frequency of the diffracted light will be downshifted from that of the incident light by an amount equal to the frequency of the sound field. The Bragg diffracted component propagates to the desired image plane where the inverse Fourier transform operation can be applied to all such components to determine the scalar field across that plane.

In this paper we will consider two cases, both involving imaging using a wedge-shaped laser beam with a vertical apex. We will assume that the height of the wedge is infinite. We can call this an infinite vertical line source (focus) of light. In the first case we will consider the object to be an infinite line scatterer (and therefore the equivalent of an infinite line source of sound) oriented vertically (i.e., parallel to the line source of light). The second case will be concerned with an infinite line source of sound oriented horizontally.

Case I: Parallel Infinite Sources

Figure 3 illustrates the physical orientation of the sources. Since both sources radiate equally in any horizontal plane this case reduces to a two-dimensional interaction. The origin of the coordinate system is assumed to be located at the line focus of the light. The sound source, extending vertically, intersects the x-y plane at $(-x_0,-y_0)$.

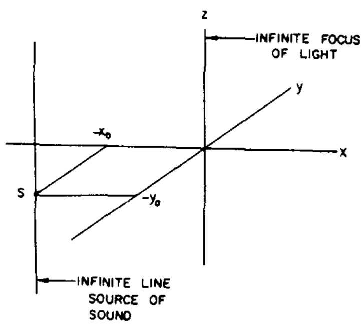

Fig. 3 - Geometry of Case I: Line Source of sound parallel to the line of focus for incident light.

Figure 4 shows two sound components interacting with two light components to produce image point S' corresponding to object point S. Each of the interacting components is oriented as shown in Fig. 4, and the resulting interaction produces a diffracted component. The intersection of the diffracted components determines the image point S'.

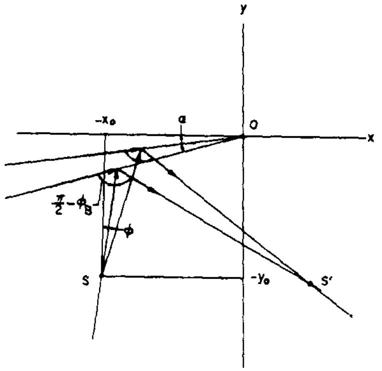

Fig. 4 - Rays of sound and light which satisfy the Bragg condition for Case I. Two typical rays of sound are shown interacting with two rays of incident light, with the result that the two diffracted rays meet at the image point (Point S').

If the sound source is infinitely long, the phase of each component of sound will be constant; i.e., each plane-wave component leaves the source with the same phase. (For simplicity, we can assume that this phase is zero.) We are interested in the relative phase of the corresponding diffracted light components at the image point. If these components are in phase with each other, then the image can be considered as unaberrated. If the phase is different for each component, then the image will be aberrated and image distortion will result, depending on the magnitude of the aberration.

For the purposes of this investigation we will reference the interaction to the sound source point S (i.e., we will consider the interaction of the plane waves at that point). Figure 5 shows two plane waves at the source point: the plane wave of the incident light and the plane wave of the diffracted light. The plane wave of the sound component that interacts with the incident light is not shown since its phase, being zero at the sound source point, does not contribute to the phase of the diffracted light. The phase of the incident light at the interaction point can be computed with reference to the line focus. If the incident light components are also assumed to have zero phase at this line focus (in a manner analogous to the sound source), the phase of the plane wave at the interaction point is equal to $-2\pi r_0/\lambda_0$ where r_0 is the distance along the propagation vector from the line focus to the plane-wave front which passes through the sound source point (as indicated in Fig. 5) and λ_0 is the wavelength of the incident light.

The phase of the diffracted wave is given by the difference in phase between the incident light component and the interacting sound component. Since the sound phase is zero, the diffracted plane wave through the source point will have the same phase as the incident light plane wave. The diffracted light then propagates a distance r_1 to the image point with a propagation constant given by

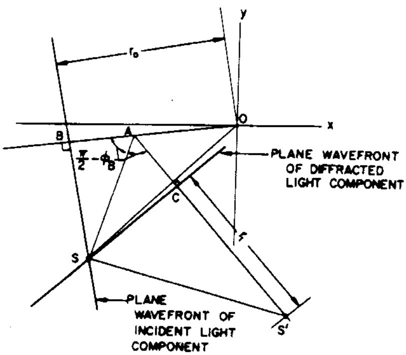

Fig. 5 - Diagram to define variables used in the calculation of the relative phase of the spatial frequency components for both the sound and the light.

$2\pi/\lambda_-$ where λ_- is the wavelength of light at the downshifted frequency. Hence the total phase β at the image point is given by

$$\beta = 2\pi\left(\frac{r_1}{\lambda_-} - \frac{r_o}{\lambda_o}\right)$$

where, in general, r_1 and r_0 depend upon the angle, ϕ, that the sound component under consideration makes with respect to the +y direction.

For the two-dimensional case, the distances r_0 and r_1 are equal. To prove this we first show that the points O and S' are equidistant from S. We do this by considering two specific interactions. The first, depicted in Fig. 6, is that of the sound component that passes through the origin O.

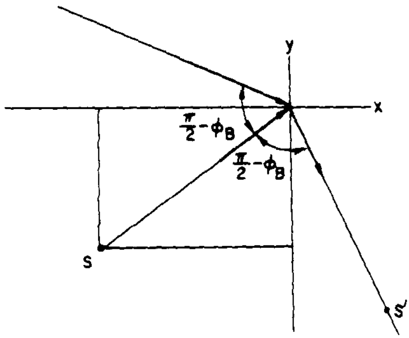

Fig. 6 - Geometry of the interacting rays when the interacting sound ray passes through the line of focus for the incident light.

A light component interacts with it and the resulting diffracted component is oriented at an angle $\pi/2-\phi_B$ with respect to the sound component. The image point S' is located at some unknown position along the propagation path of the diffracted component. The second special interaction to be considered is that of the incident light component which passes through the sound source point S as shown in Fig. 7. A sound component interacts with this light component and produces a diffracted component. Again the image point lies somewhere

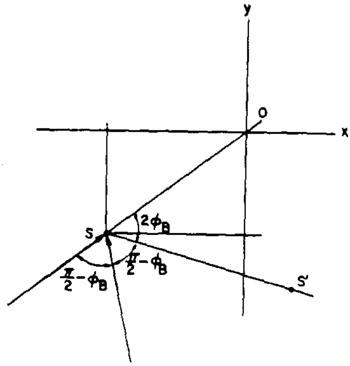

Fig. 7 - Geometry of the interacting rays when the interacting light ray passes through the sound source (Point S).

along the propagation path. The angle OSS' is equal to $2\phi_B$ as is apparent from simple geometry. By superimposing the propagation paths for the diffracted waves of the two cases, as shown in Fig. 8, we locate the image point S' which, by definition, is the intersection of the two paths. The angle OS'S is equal to $\pi/2-\phi_B$ and hence the triangle OSS' is an isoceles triangle. O and S' are therefore equidistant from S.

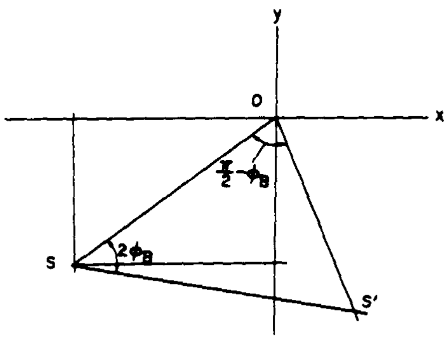

Fig. 8 - Relative position of sound source, S image point, S, and line of focus for incident light, O.

Referring to Fig. 5, we note that the point S lies on the bisector of the angle BAC. This implies that the normals SB and SC are of equal length. In Fig. 5 the right triangle SBO has two of its sides equal to two sides of the right triangle SCS'. Hence the third sides of each must be equal (i.e, OB = CS'). This is equivalent to saying that $r_0 = r_1$, proving that the propagation distances of both the incident and diffracted waves are equal. Thus we can write the expression for the phase as

$$\beta = 2\pi r_o\left(\frac{1}{\lambda_-} - \frac{1}{\lambda_o}\right)$$

It can be shown geometrically[5] that a sound component propagating at an angle ϕ with respect to the +y direction will interact with a light component that makes an angle $\alpha = \phi+\phi_B$ with respect to the +x direction. This implies, as in Fig. 9, that the distance r_0 is given by

$$r_o = \sqrt{x_o^{\,2}+y_o^{\,2}} \quad \cos(\xi-\alpha)$$

$$r_o = \sqrt{x_o^{\,2}+y_o^{\,2}} \quad \cos(\tan^{-1}\frac{y_o}{x_o} - \phi - \phi_B)$$

$$r_o = d\,\cos(\phi+\phi_B - \tan^{-1}\frac{y_o}{x_o})$$

where d is the distance of the sound source point from the origin. The phase of the diffracted components is then given by

$$\beta = 2\pi\left(\frac{1}{\lambda_-} - \frac{1}{\lambda_o}\right) d\,\cos(\phi+\phi_B - \tan^{-1}\frac{y_o}{x_o})$$

$$\beta = 2\pi\left(\frac{f_s}{f}\,\frac{1}{\lambda_o}\right) d\,\cos(\phi+\phi_B - \tan^{-1}\frac{y_o}{x_o})$$

where f_s is the sound frequency and f is the incident light frequency.

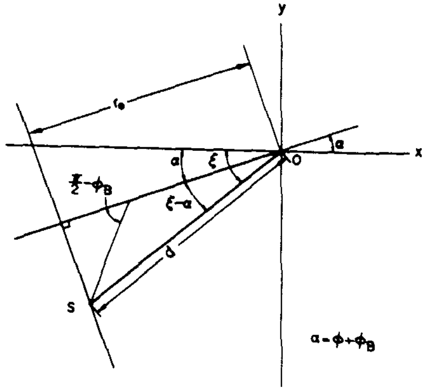

Fig. 9 - Diagram to determine the distance r_0 in terms of the position (x_0, y_0) of the sound source, S.

We note from this expression that there is a phase variation of the diffracted components at the image point. This variation is expected to be small since the difference between λ_0 and λ_- is very small. If it were not for the shift in frequency in the interaction, the phase terms would cancel and the image would be unaberrated. Substituting some typical values into the expression ($d=10$ cm; $f_s=10$ MHz; $f=10^{14}$ Hz; $\lambda_0=0.6\times10^{-4}$ cm) the maximum excursion of the phase is seen to be on the order of 10^{-4} radians. Thus, in this case, we can assume that image distortion is extremely small.

Experimental results[6] verify that there is little or no aberration for the case of a vertical sound source. It was predicted theoretically that the resolution for objects with this orientation would be determined by the Rayleigh criterion where the numerical aperture is taken to be the sine of the semi-angle of the light wedge.[7] This relation proved to be experimentally correct.[6] There was no indication in the experiments that the resolution was substantially limited by image aberrations. With the particular Bragg imaging system used, the theoretically predicted resolution limit was 5Λ. The experimentally observed value was 10Λ, a difference attributable to experimental error and other factors (such as noise) rather than to phase aberrations.

Case II: Vertical Light Source, Horizontal Sound Source

Figure 10 illustrates the orientation of the infinite sources for this second case. The infinite line source of sound is now located in the horizontal plane. The x axis is assumed to be parallel to the sound source. Consider the point $(-x_0,-y_0)$ located on the sound source. As noted in Fig. 10 the interaction geometry is more complicated and cannot be reduced to two dimensions. Here the image is formed by components which are not coplanar. The interaction occurs between a sound component inclined at angle θ above the x-y plane and a light component with a propagation direction which makes an angle α with respect to the x direction. By symmetry one could imagine a similar interaction with a sound component that declined from the x-y plane at an angle $-\theta$. The diffracted light component would be placed symmetrically and the image point would be where these propagation paths cross in the x-y plane. For small values of θ Korpel[8] showed that the diffracted components form an image point that is located a distance $(\Lambda/\lambda)y_0$ away from the source point in a direction parallel to the x axis (i.e., the image point is located at the coordinates $(-x_0+(\Lambda/\lambda)y_0, -y_0)$). Although the image point for sound components of relatively large angles of inclination (greater than angles of about 45 degrees) moves away from this small-angle image point, this latter point offers a convenient position to measure the phase aberrations. The fact that there are other components whose propagation paths do not intersect at this point will result in variations in the phase computed at this point for these components.

Again the measurement of the phase will be referenced to the sound-source point and the phase of the incident light component will be specified

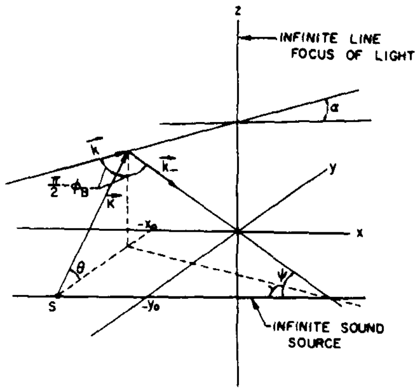

Fig. 10 - Geometry of Case II: Line source of sound oriented at right angles to the line of focus for incident light.

by minus the propagation constant, $2\pi/\lambda_0$, times r_0, the distance between the line focus and the intersection of the plane wavefront that contains S and the propagation direction (as illustrated in Fig. 11). Similarly the phase contribution due to the diffracted light propagation is given by $2\pi/\lambda_-$ times r_1, the distance along the propagation direction from the wavefront that contains S to the image point. In this case both r_0 and r_1 are functions of θ and must be computed geometrically.

In this special case it has been shown that the angle α and θ must obey the following relation in order for interaction to occur:

$$\sin = \frac{\sin\phi_B}{\cos\theta}$$

As depicted in Fig. 11, the expression for r_0 is

$$r_o = (y_o - x_o \tan\alpha)\sin\alpha + \frac{x_o}{\cos\alpha}$$

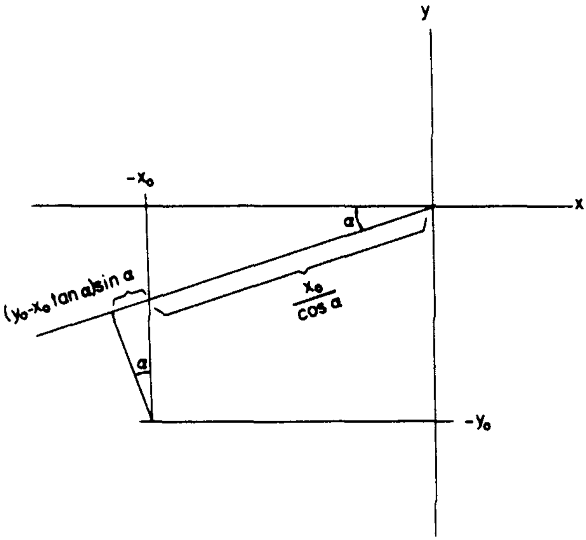

Fig. 11 - Diagram to show the normal distance from the origin of a line in the x-y plane passing through point x_o, y_o.

Upon trigonometric substitution this becomes

$$r_o = x_o \cos\alpha + y_o \sin\alpha$$

which in turn can be expressed as,

$$r_o = x_o \sqrt{1-\left(\frac{\sin\phi_B}{\cos\theta}\right)^2} + y_o \frac{\sin\phi_B}{\cos\theta}$$

To calculate r_1 we first consider the image point to be at arbitrary coordinates (x,y) and obtain a general formula. We will then substitute the value of the image point discussed above. Figure 12 shows the x-y plane and the projections of two wavefronts: one passing through the source point and the other passing through the image point. As shown in the figure, the distance between these projections, r_{xy}, is

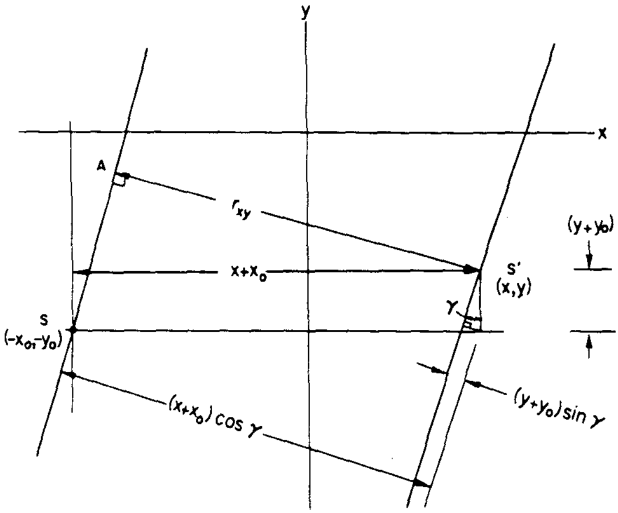

Fig. 12 - Diagram which shows the intersection of two parallel wavefronts with the x-y plane such that one wavefront passes through the source point, S, and the other through the image point, S'.

$$r_{xy} = (x+x_o)\cos\gamma-(y+y_o)\sin\gamma$$

where γ is the azimuth angle of the direction of propagation. The azimuth angle is here defined as the angle that the projection (on the x-y plane) of the propagation direction makes with respect to the +x axis.

The distance r_{xy}, as seen in Fig. 13, is simply related to r_1. Thus

$$r_1 = r_{xy}\cos\psi$$

where ψ is the inclination of the diffracted component with respect to the x-y plane. It has been shown that the azimuth angle of the propagation direction is given by

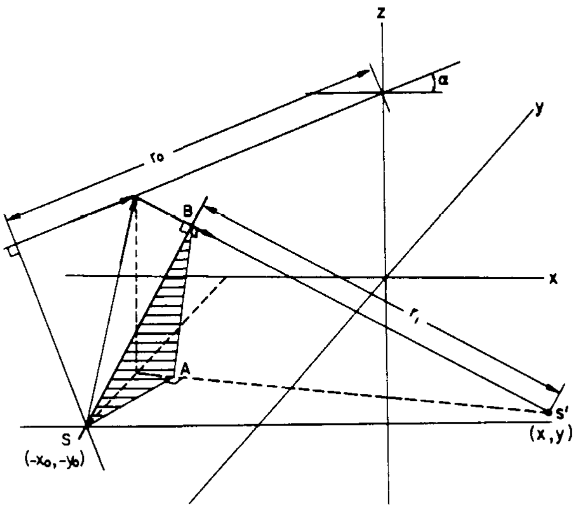

Fig. 13 - Pictorial view of plane-wave propagation distances. r_0 is the distance a plane wave in the incident light will travel in going from source point S to the z axis. r_1 is the distance a plane wave will travel in going from point S to S'.

$$\sin\gamma = \frac{\sin\phi_B \cos 2\theta}{\cos\theta \cos\psi}$$

and that the inclination angle is given by

$$\sin\psi = \frac{\lambda}{\Lambda} \sin\theta$$

This gives a propagation distance as follows:

$$r_1 = \frac{\sin\phi_B \cos 2\theta}{\cos\theta}$$

$$\left[(y+y_o)+(x+x_o)\sqrt{\left(\frac{\cos\theta}{\sin\phi_B \cos 2\theta}\right)^2 -1}\right]$$

If we now assume that the image point is located at $(-x_0+(\Lambda/\lambda)y_0, -y_0)$ as described before, then this expression reduces to

$$r_1 = \frac{y_o}{2\sin\phi_B}\sqrt{1-\left(\frac{\sin\phi_B \cos 2\theta}{\cos\theta}\right)^2}$$

and the total phase of the incoming components as a function of the sound component inclination angle θ is given by

$$\beta = 2\pi\left(\frac{r_1}{\lambda_-} - \frac{r_o}{\lambda_o}\right)$$

$$\beta = 2\pi\,\frac{y_o}{2\lambda_-\sin\phi_B}\sqrt{1-\left(\frac{\sin\phi_B\cos 2\theta}{\cos\theta}\right)^2}$$

$$-\frac{x_o}{\lambda_o}\,\frac{\sin\phi_B}{\cos\theta} - \frac{y_o}{\lambda_o}\sqrt{1-\left(\frac{\sin\phi_B}{\cos\theta}\right)^2}$$

This is the complete expression for the phase at the assumed image point. We have computed β as a function of θ numerically on a digital computer for some typical input values (x_0=6.0 cm; y_0=3.0 cm, f_s=17.0 MHz; λ_0= 0.6328 μm). The results are shown in Fig. 14. Here it is seen that the phase for sound components with inclination angle greater than about 40 degrees changes rapidly and gives rise to severe distortion in the image. This distortion would adversely effect the resolution capability of the image for features with this orientation.

For a horizontally oriented sound source, experimental results support the conclusion that phase aberrations significantly restrict the resolvable detail.[6] Theory predicts that the minimum resolvable distance will be inversely proportional to the sine of the maximum inclination angle made by any sound component which interacts with the incident light. However, in experiments in which

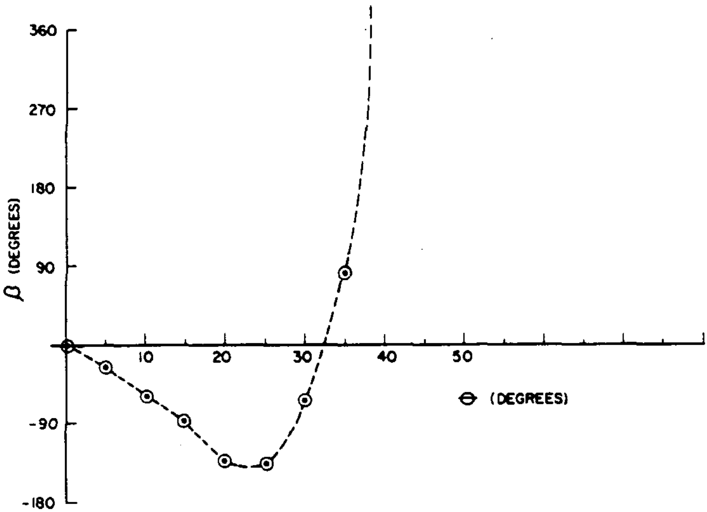

Fig. 14 - The phase angle, β, for rays involved in the imaging of wires oriented at right angles to the line of focus of the incident light.

this angle approached 90 degrees, the actual resolution observed corresponds to an "effective" maxinum angle of only about 50 degrees. The minimum resolvable detail for this case was 2/3 Λ. The "effective" aperture, as limited by phase aberrations, for a system having the hypothetical dimensions used in the computer calculation would be that corresponding to a maximum "effective" inclination of only about 40 degrees, a number compatible with the experimental result cited above.

SUMMARY

This paper has considered phase aberrations in Bragg imaging systems. The aberrations were studied by computing the phase variation of the

spatial components as they arrive at the image location for an infinite line source of sound. An unaberrated image would be one where all the components arrive in phase with each other. For an out-of-phase situation, the severity of the aberration is determined by the relative deviation from the ideal unaberrated case.

Two orientations of the sound source were investigated: one vertical and the other horizontal. For the vertical orientation, the phase aberrations were found to be insignificant. There is only an extremely small variation in the phase of the incoming components. However, for a horizontally oriented sound source where the sound components in general are sharply inclined with respect to the horizontal plane, the aberrations were found to be substantially more severe. Under these conditions, the aberrations place an effective limitation on the attainable resolution in the image. Experimental results are compatible with the predictions of the above theory.

ACKNOWLEDGEMENTS

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